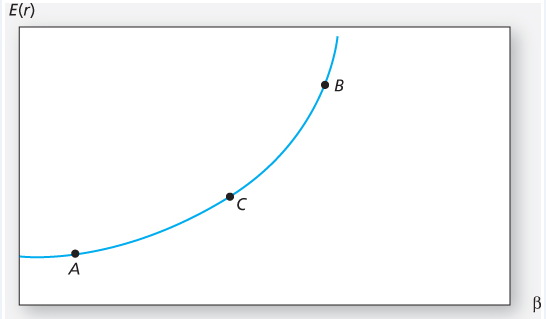
**Investments Problem Sheet 4 Lent Term 2024**

**True-False**

1. Arbitrage Pricing Theory says that if there is no arbitrage then all share prices must be driven by a small number of factors.
2. If a substantial body of investors decided to exclude “sin stocks” (eg tobacco, armaments) from their portfolios, both the Arbitrage Pricing Theory (APT) and the Capital Asset Pricing Model (CAPM) would continue to hold.
3. If expected return rises with beta as shown in the following figure, there is a clear arbitrage opportunity.



4. Consider a single factor APT. Portfolio A has a beta of 1.0 and an expected return of 12%. Portfolio B has a beta of 1.5 and an expected return of 17%. The risk-free rate of return is 4%. If you wanted to take advantage of an arbitrage opportunity, you should take a short position in portfolio A and a long position in portfolio B.

**Questions**

1. You are working for an investment house that believes in the APT. They use a four factor model, as in Chen, Roll and Ross. They have constructed four portfolios that are maximally correlated with each of the four factors (unexpected changes in inflation, slope of the term structure, corporate risk premium and output). They have estimated factor betas for a number of shares, and you are particularly interested in three of the shares (A, B and C). Their betas are given below:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Share | Factor1 | Factor 2 | Factor 3 | Factor 4 |
| A | +0.5 | +1.4 | -0.2 | +1.0 |
| B | -0.8 | +2.0 | +0.5 | +0.7 |
| C | +3.1 | +0.2 | -1.6 | +1.6 |

1. Given that the risk free rate is 5% and the risk premium on the four factors is estimated at 2.0%, 1.0%, 1.5% and 1.0%, what is the expected return on each of the shares?
2. What are the betas of a portfolio that is invested two thirds in B and one third in C? What is its expected return? How does the portfolio differ from one that is invested fully in A?

2. Suppose there are two independent factors F1 and F2 . All stocks have independent firm-specific components with a standard deviation of 45%. The following are well-diversified portfolios.

|  |  |  |  |
| --- | --- | --- | --- |
| Portfolio | Beta on F1 | Beta on F2 | Expected Return |
| A | 1.5 | 2 | 31% |
| B | 2.2 | -0.4 | 26% |
| C | 0 | 0 | 6% |

What is the expected return-beta relationship in this economy?

3. The following annual excess rates of return were obtained for nine individual stocks and a market index.

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Year | Market Excess Returns(%) | A | B | C | D | E | F | G | H | I |
| 1 | 29.65 | 33.88 | -25.20 | 36.48 | 42.89 | -39.89 | 39.67 | 74.57 | 40.22 | 90.19 |
| 2 | -11.91 | -49.87 | 24.70 | -25.11 | -54.39 | 44.92 | -54.33 | -79.76 | -71.58 | -26.64 |
| 3 | 14.73 | 65.14 | -25.04 | 18.91 | -39.86 | -3.91 | -5.69 | 26.73 | 14.49 | 18.14 |
| 4 | 27.68 | 14.46 | -38.64 | -23.31 | -0.72 | -3.21 | 92.39 | -3.82 | 13.74 | 0.09 |
| 5 | 5.18 | 15.67 | 61.93 | 63.95 | -32.82 | 44.26 | -42.96 | 101.67 | 24.24 | 8.98 |
| 6 | 25.97 | -32.17 | 44.94 | -19.56 | 69.42 | 90.43 | 76.72 | 1.72 | 77.22 | 72.38 |
| 7 | 10.64 | -31.55 | -74.65 | 50.18 | 74.52 | 15.38 | 21.95 | -43.95 | -13.40 | 28.95 |
| 8 | 1.02 | -23.79 | 47.02 | -42.28 | 28.61 | -17.64 | 28.83 | 98.01 | 28.12 | 39.41 |
| 9 | 18.82 | -4.59 | 28.69 | -0.54 | 2.32 | 42.36 | 18.93 | -2.45 | 37.65 | 94.67 |
| 10 | 23.92 | -8.03 | 48.61 | 23.65 | 26.26 | -3.65 | 23.31 | 15.36 | 80.59 | 52.51 |
| 11 | -41.61 | 78.22 | -85.02 | -0.79 | -68.70 | -85.71 | -45.64 | 2.27 | -72.47 | -80.26 |
| 12 | -6.64 | 4.75 | 42.95 | -48.60 | 26.27 | 13.24 | -34.34 | -54.47 | -1.50 | -24.46 |

Suppose that in addition to the market factor that has been considered, a second factor is considered. The values of this factor for years1 to 12 were as follows:

|  |  |
| --- | --- |
| Year | % Change in Factor Value |
| 1 | -9.84 |
| 2 | 6.46 |
| 3 | 16.12 |
| 4 | -16.51 |
| 5 | 17.82 |
| 6 | -13.31 |
| 7 | -3.52 |
| 8 | 8.43 |
| 9 | 8.23 |
| 10 | 7.06 |
| 11 | -15.74 |
| 12 | 2.03 |

1. Perform the first-stage time-series regressions and tabulate the relevant summary statistics (Hints: use a multiple regression as in a standard spreadsheet package. Estimate the betas of the 12 stocks on the two factors).
2. Specify the hypothesis for a test of a second-stage regression for the two-factor Security Market Line.
3. Do the data suggest a two-factor economy?